CECS 329, Quiz 3 Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS.

- For each LO, write your solutions to all parts using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet. Use **SEPARATE SHEETS** for different LO problems.
- You may solve an amount of problems that sums to **AT MOST THREE WHOLE PROBLEMS**. Two parts of different LO's count for one whole problem. For example, solving LO1 part a, LO2 part b, LO7 part b, LO4 part a, and LO5 counts as three whole problems. Further solutions that go beyond this amount will be ignored.

LO1. Do the following.

(a) Is it true that $\{1, 3, 5\} \in \{1, 2, 3, 4, 5\}$? **Explain**. Also, if function

$$\text{digits} : \mathcal{N} \rightarrow \mathcal{P}(\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\})$$

is defined by digits(n) equals the set of all digits that occur in natural number n, then evaluate digits on input 7938493. Hint: to receive credit you must demonstrate the use of both function and set notation.

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(\overline{x}_1, x_4), (x_1, \overline{x}_5), (\overline{x}_2, \overline{x}_3), (\overline{x}_2, x_4), (x_2, x_6), (x_3, x_4), (\overline{x}_3, x_6), (\overline{x}_4, \overline{x}_5), (\overline{x}_4, x_5)\}.$$

i. Draw the implication graph $G_{\mathcal{C}}$.

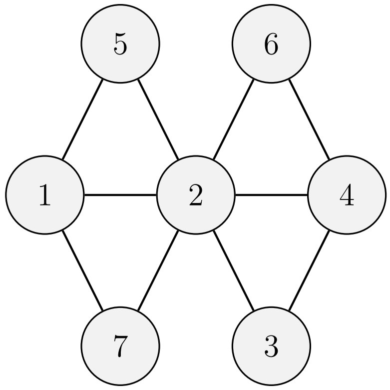
ii. Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

LO2. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Vertex Cover` $\rightarrow$ `Half Vertex Cover`, draw $f(G, k)$ for the `Vertex Cover` instance whose graph is shown below, and for which $k = 2$.

(c) For the mapping reduction f : `Vertex Cover` $\rightarrow$ `Half Vertex Cover`, draw $f(G, k)$ for the `Vertex Cover` instance whose graph is shown below, and for which $k = 3$.



(d) Verify that both (G, k) and $f(G, k)$ are either both positive instances, or are both negative ones. Explain and show work.

LO3. Answer the following. Note: a minimum total of 15 points must be scored in order to pass.

(a) An instance of `Set Packing` is a positve integer $k > 0$ and a collection of sets $\mathcal{S}$, where each set $C \in S$ is a subset of $\{1, \ldots, n\}$ for some $n \geq 1$. The problem is to decide if there are k sets $C_1, \ldots, C_k$ in $\mathcal{S}$ that are **pairwise disjoint** meaning that, for every $1 \leq i < j \leq n$, $C_i \cap C_j = \emptyset$. Which of the following describes a natural certificate for `Set Packing`? (7 pts)

i. an array a of length k, where $a[i] \in \mathcal{S}$, for $i = 1, \ldots, k$

ii. a subset C that is a member of $\mathcal{S}$ and for which $|C| = k$

iii. an array a of members from $\{1, \ldots, n\}$

iv. a subset of $\mathcal{S}$ that is neatly packed

(b) Provide size parameters for `Set Packing`. Hint: there are two of them (6 pts)

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of the `HP` decision problem is a simple graph $G = (V, E)$ and the problem is to decide if G has a Hamilton path.

ii. An instance of `Longest Path` is a tree (i.e. a connected simple graph have no cycles) T and an integer $k \geq 0$, and the problem is to decide if there is a path in T whose lenth is at least k.

iii. An instance of the `Unfallible` decision problem is a Boolean formula F and the problem is to decide if there are no assignments α to the variables of F for which $F(\alpha) = 0$.

iv. The `2SAT` decision problem

LO4. Answer the following.

(a) Consider the instance of `3SAT` instance

$$\mathcal{C} = \{c_1 = (\overline{x}_1, \overline{x}_3, x_4), c_2 = (x_2, \overline{x}_3, x_4), c_3 = (\overline{x}_2, \overline{x}_3, \overline{x}_4), c_4 = (x_1, x_2, \overline{x}_3))\}$$

and the mapping reduction f from `3SAT` to `Clique`. Answer the following with respect to $f(\mathcal{C}) = (G, k)$.

i. How many vertices and edges does G have? Explain and Show work.

ii. What is the value of k? Explain.

iii. Verify that G has a k-clique. For each vertex in the clique, indicate the vertex group to which it belongs. What does this clique tell you about $\mathcal{C}$?

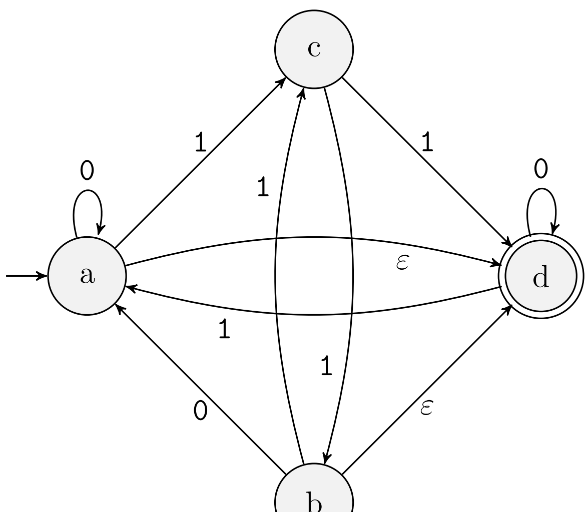
(b) Given the Boolean formula $F = x_1 \wedge (\overline{x}_2 \vee x_3)$, do the following. Provide the corresponding Boolean formula that is satisfiability-equivalent to F and is the starting point of the Tseytin transformation. Hint: the new formula has both x and y variables. Show how Tseytin converts the *first* double-arrow subformula (of the formula you wrote) into a set of `3SAT` clauses.

LO5. Do the following.

(a) All words that either start with a 1 and have odd length, or (inclusive) start with a 0 and have at least one other 0 and at most one 1.

(b) Demonstrate the computation of M on inputs i) $w_1 = 11001$ and ii) $w_2 = 001010$. For each computation, indicate whether w is accepted or rejected.

LO6. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram. (8 pts)

(c) Show the computation of M on input $w = 11001$.

LO7. Do the following.

(a) Let L_1 denote the language of binary words of length four that have an odd number of 1's. Let L_2 denote the language consisting of all binary words w having length four and ending with a 0. Use set notation to write the members of $L_1 \oplus L_2$. (6 pts)

Solution. We have

$$L_1 \oplus L_2 = \{0001, 0111, 1011, 1101, 0000, 1100, 1010, 0110\}.$$

(b) Let L denote the language of binary words that begin with two 1's and have an odd number of 0's. Is 11001001101011001100 a member of L^*? Justify your answer.

Solution. 11001001101011001100 is not a member of L^* since it begins with 11 and, whenever we append and odd number of 0's to those two 1's, then the remainder of the word does not begin with 11, and so cannot be parsed into subwords that belong to L. Therefore, the word can never be parsed as uv where $u \in L$ and $v \in L^*$ and so the entire word is not a member of L^*.

(c) Provide a regular expression that describes all nonempty binary words that contain an even number of 0's or (inclusive) contain exactly two 1's that are separated by an odd number 0's (and may have additional 0's to the left and right of the two 1's).

Solution.

$$1^*(01^*01^*)^*1^* \cup 0^*10(00)^*10^*.$$