CECS 329, Quiz 4 Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS.

- For each LO, write your solutions to all parts using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet. Use **SEPARATE SHEETS** for different LO problems.
- You may solve any number of problems, but please be aware that the quiz ends at 8:45 am. Also, extra credit will be awarded for the complete passing of either LO8 or LO9 (or both!).

LO9. Do the following.

(a) Use the CFG rules below to derive the expression $a \times (b + a)$. Make sure to use a left-most deriviation and replace at most one variable in each step.

$$E \rightarrow E + T \mid T$$

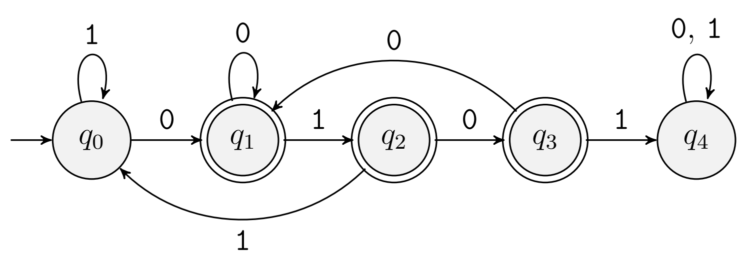
$$T \rightarrow T \times F \mid F$$

$$F \rightarrow (E) \mid a \mid b$$

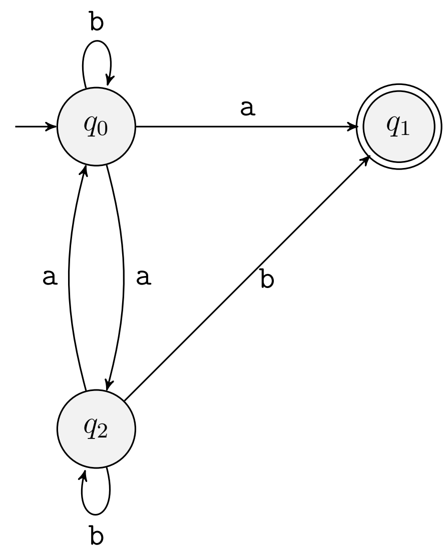
(b) Provide the rules for a context-free grammar whose terminal set is $\Sigma = \{a, b, c\}$ and which derives exactly those words that are palindromes (i.e. read the same forwards as backwards), including the empty word ε.

LO8. Do the following.

(a) Modify the NFA N shown below, so that the resulting NFA N' accepts L^*, where L is the language accepted by N.



(b) Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes the language accepted by the NFA shown below. Redraw the NFA after each step (i.e. state removal).



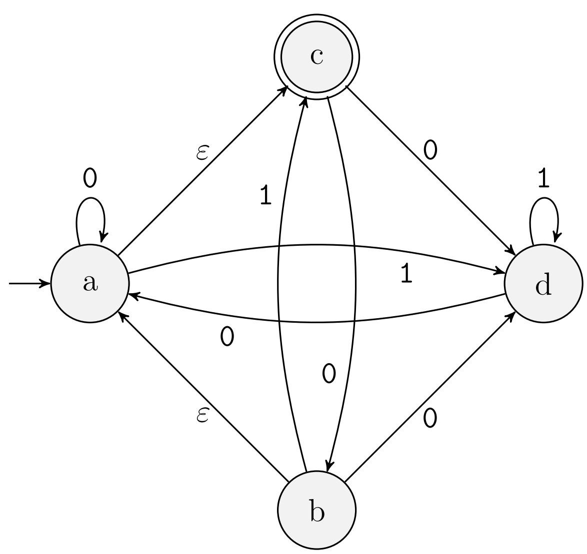
LO7. Do the following.

(a) Let L_1 denote the language of binary words of length four that have exactly two 1's. Let L_2 denote the language consisting of all binary words w having length four and ending with a 1. Use set notation to write the members of $L_1 \oplus L_2$.

(b) For L_1 and L_2 defined in part a, is it true that $01011100 \in L_2 \circ L_1$? Explain.

(c) Provide a regular expression that describes all binary words that begin with a 1, end with a 0, and have an even number of 1's.

LO6. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram. (8 pts)

(c) Show the computation of M on input $w = 11001$.

LO5. Do the following.

(a) Provide the state diagram of a DFA that accept exactly those words that either have an even number of 0's or (inclusive) have exactly two 1's.

(b) Demonstrate the computation of M on inputs i) $w_1 = 11001$ and ii) $w_2 = 101010$. For each computation, indicate whether w is accepted or rejected.

LO4. Answer the following.

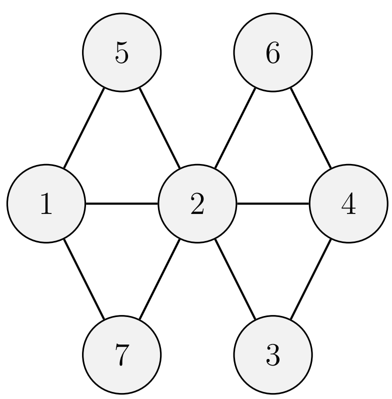
(a) Consider the mapping reduction $f : \texttt{3SAT} \rightarrow \texttt{Subset Sum}$ from `3SAT` to `Subset Sum` that was presented in lecture and the `3SAT` instance

$$\mathcal{C} = \{c_1 = (\overline{x}_1, \overline{x}_2, \overline{x}_3), c_2 = (\overline{x}_1, x_2, x_3), c_3 = (x_1, \overline{x}_2, \overline{x}_3)\}.$$

i. Compute $f(\mathcal{C}) = (S, t)$ by providing the full table that shows the members of S and the target value t.

ii. Verify that f is valid for input $\mathcal{C}$ in the sense that both $\mathcal{C}$ and $f(\mathcal{C}) = (S, t)$ are both positive instances. Do this by providing a satisfying assignment α and use it to determine a subset $A \subseteq S$ that sums to t.

(b) Let G be the graph shown below. Compute $f(G)$, where $f : \texttt{HC} \rightarrow \texttt{TSP}$ is the mapping reduction from `Hamilton Cycle` to `Traveling Salesperson` that was provided in lecture.



LO3. Answer the following. Note: a minimum total of 15 points must be scored in order to pass.

(a) An instance of `Quadratic Congruences` is a triple of positive integers (a, b, c) and the problem is to decide if there is a positive integer $x < c$ for which $x^2 \bmod b = a$. Which of the following describes a natural certificate for `Quadratic Congruences`? (7 pts)

i. a positive integer c

ii. a positive integer x

iii. a positive integer a

iv. a positive integer b

(b) Provide the single size parameter for `Quadratic Congruences`. (6 pts)

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of `Fallible` is a Boolean formula F. The problem is to decide if there an assignment that can be made to the variables of F so that F evaluates to 0.

ii. An instance of `Mine Sweep` is a simple graph $G = (V, E, f)$ where $f : V \to \{-1, 0, 1, \ldots\}$ is a function from the set of vertices to the set of natural numbers, including -1. If $f(v) \geq 0$, then it means that a total of $f(v)$ neighbors of v (i.e., vertices that are adjacent to v) must have a mine placed on them. On the other hand, if $f(v) = -1$, then there is no constraint on how many neighbors of v must have a mine. The problem is to decide if there is a function $g : V \to \{0, 1\}$, such that i) $g(v)$ indicates whether or not a mine is placed on vertex v, and ii) for all $v \in V$, if $f(v) \geq 0$, then

$$f(v) = \sum_{u \in N(v)} g(u),$$

where $N(v)$ is the set of all neighobrs of v. In other words, function g meets all the mine constraints that are indicated by f.

iii. An instance of `Palindrome` is an array a of integer, and the problem is to decide if a reads the same forwards as backwards, meaning that, for all $i \in \{0, 1, \ldots, n-1\}$, $a[i] = a[n-1-i]$.

iv. An instance of `Large Vertex Covers` is a graph $G = (V, E)$, and the problem is to decide if every vertex cover of G has size at least $n/2$, where $n = |V|$.

LO2. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Subset Sum` $\to$ `Set Partition`, provide $f(S, t)$ for the `Subset Sum` instance with $S = \{8, 9, 12, 15, 22, 27\}$ and $t = 36$.

(c) Verify that the reduction is valid for (S, t) in that both (S, t) and $f(S, t)$ are either both positive or negative instances. Justify your answer.

LO1. Do the following.

(a) Is it true that $\{1, 3, 5\} \subseteq \{1, 2, 3, 4, 5\}$? **Explain**. Also, if function

$$\text{letters} : \{\text{a,b,c,d,e,f,g,h,i}\}^* \to \mathcal{P}(\{\text{a,b,c,d,e,f,g,h,i}\})$$

is defined by letters(w) equals the set of all letters that occur in word w, then evaluate `letters` on input $w =$ beige. Hint: to receive credit you must demonstrate the use of both function and set notation.

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (x_2, x_3), (\overline{x}_1, x_3), (\overline{x}_4, x_5), (\overline{x}_4, \overline{x}_5), (\overline{x}_3, x_4)\}.$$

i. Draw the implication graph $G_{\mathcal{C}}$.

ii. Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.