

CECS 528, In Class Assignment 1, January 30th, 2026, Dr. Ebert

Directions.

1. Form a group with two other students and solve each of the following three problems.
2. Each student is responsible for i) handwriting a solution to one of the problems on a separate sheet of paper, and ii) providing feedback to the other two students regarding their solutions.
3. In addition to the solution, each page should also include i) the problem number, ii) name of the author, and iii) signatures from the other two students. By signing the page, each of the other two students are certifying that they either approve of the solution and/or have offered constructive feedback to the author.
4. All three pages should be turned in before leaving class. Please do *not* staple them, as they will be graded and returned to each author at the next meeting.

Problems

1. This problem is inspired by my supervisor at Arcadia Design Systems who told me “linear and log-linear [algorithms]: good. Anything else, be very careful how you use it”. Suppose that a computer’s cpu is capable of executing a single instruction in 5×10^{-10} seconds. The plan is to run a time-intensive program on the computer for 30 days. For each of the following scenarios determine the largest problem size n that can be solved during this time. Show all steps for full credit. Please round up each answer to the nearest integer. All answers should be accurate to the nearest integer.
 - a. The program solves instances of **Matrix Addition** and requires about $100n^2$ instructions to add two $n \times n$ matrices. (5 pts)
 - b. The program solves instances of **Matrix Multiplication** and requires about $180n^3$ instructions to multiply two $n \times n$ matrices. (5 pts)
 - c. The program solves instances of the **3SAT** logic problem and implements the best known **3SAT** algorithm which requires about $50 \left(\frac{4}{3}\right)^n$ instructions to solve a **3SAT** instance that depends on n variables. (5 pts)
2. Stassen’s matrix multiplication divide-and-conquer algorithm is capable of multiplying two $n \times n$ matrices using $T(n)$ steps, where $T(n)$ satisfies the recurrence

$$T(n) = 7T(n/2) + O(n^2).$$

- a. Use the Master Theorem to determine the growth of $T(n)$. Indicate which case is being used, show work, and provide an appropriate ϵ value if needed by that case. (5 pts)

- b. Use the function growth computed for $T(n)$ in part a along with the assumption that each step requires about 250 clock cycles when executed by the computer described in Problem 1. Determine the largest matrix dimension n that can be solved by the computer using Strassen's algorithm and running for up to 30 days. Compare your answer with the answer to Problem 1b. (5 pts)
3. If $T(n)$ satisfies the recurrence

$$T(n) = 2T(n/2) + n \log^2 n,$$

then use the substitution method to prove that $T(n) = \Omega(n \log^2 n)$. (10 pts)