

Student Names:

Problems

1. Complete Example 10 from the Dynamic Programming lecture by demonstrating the Matrix Chain Multiplication algorithm. On a single collective sheet, complete each cell of the 5×5 matrix. Make each cell large enough to fit i) the subproblem solution, ii) the k value used for the solution, and iii) the initials of the student who solved that subproblem. Each student should show their work on the same sheet. Divide the work evenly so that each student should solve at least one subproblem from cell set $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$, one from $\{(1, 3), (2, 4), (3, 5)\}$, and one from $\{(1, 4), (2, 5), (1, 6)\}$. (15 points)
2. Suppose n packages are in a queue and are to be loaded on to a queue of delivery trucks, where package i consumes s_i units of space. The delivery company has concluded that the optimal (in terms of cost) loading scheme should be one that balances the load of each truck in terms of the amount of remaining empty space. Each truck has a space capacity of $M > 0$ units, where M is some constant. The trucks to be loaded are in a FIFO queue and, at any time, only the first truck in the queue can be loaded. The packages are also in a FIFO queue and must be loaded in that order. The goal is to provide the trucks roughly the same volume of packages, meaning that it is better to have all trucks partially full than to have trucks ahead in the queue become filled to capacity, while trucks later in the queue are almost empty. In other words, the penalty

$$\sum_{t=1}^m e_t^2,$$

should be minimized, where m is the number of trucks needed to load the packages, and e_t denotes the amount of empty space in truck t after it is loaded.

For the following problems, you may assume that there are always a sufficient number of trucks to support the optimal solution and that any unused trucks are not to be considered when computing the penalty score.

- Student A. Imagine that you know how to solve the **Package Loading** problem described above for package-queue sizes of $n - 1$ or fewer packages. Moreover, the first $n - 1$ packages have already been loaded on to the trucks. But just as all the loaded trucks are about to pull away, alas!, the n th package arrives to the queue and must be loaded. In (mostly) words, describe the possible actions that you can take with regards to adding this package to the collective shipment without breaking any of the above-described rules. (5 pts)
- Student B. Given a problem instance that consists of a sequence of positive integers s_1, s_2, \dots, s_n , along with a truck space-capacity constant M , describe a typical subproblem instance. Use this description to define a function that computes the solutions to all the subproblem instances, including the original problem instance. Provide a recursive formula for computing this function. Hint: the formula should *not* depend on the number of trucks. In other

words, we may assume that there are always enough trucks available to optimize the problem instance, and thus the number of trucks in the queue does not matter. (10 pts)

Student C. Apply your recurrence to the package sequence 3, 3, 4, 3, 5, 2, 2, with $M = 10$, where the numbers in the sequence represent the package sizes, and a package with size 3 is at the front of the queue. (5 pts)