CECS 528, In Class Assignment 7, Friday March 20th, Spring 2026, Dr. Ebert

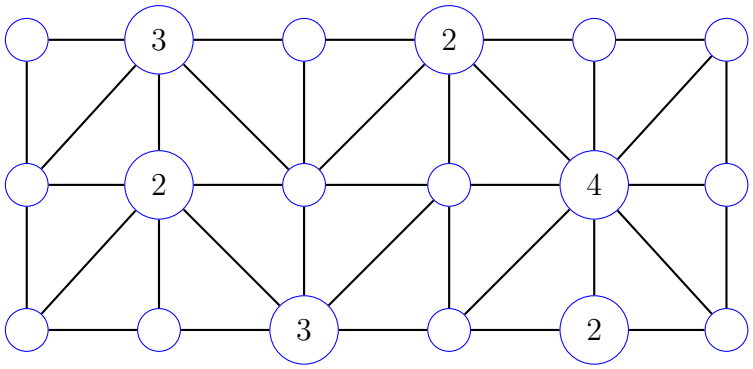
Please collectively and sequentially solve the parts to the following problem as a group and, for each part of the problem, elect a different group member to write the solution to that part.

Problem

An instance of the `MineSweep` decision problem is a simple graph $G = (V, E)$ and a map $\text{val} : V \to \mathcal{N}$ for which some of G's vertices are labeled with a positive integer. Namely, if $\text{val}(v) = 0$, then v is not labeled, but if $\text{val}(v) > 0$, then $\text{val}(v)$ gives the value of the label.

The problem is to decide if there is a way to place mines on some of G's unlabeled vertices so that, for each $v \in V$ that is labeled with some integer $k = \text{val}(v)$, exactly k of v's neighbors have been assigned a mine.

1. Show that the following labeled graph is a positive instance of `MineSweep`. Do so by placing an "x" in each vertex that should have a mine. (10 pts)



2. For a given instance $(G = (V, E), \text{val})$ of `MineSweep`, a certificate for this instance is a function $f : V \to \{0, 1\}$ for which $f(v) = 1$ iff a mine is to be placed on vertex v. Provide the pseudocode for a verifier algorithm that takes as input i) an instance $(G = (V, E), \text{val})$ of `MineSweep`, and ii) a certificate $f : V \to \{0, 1\}$, and decides if the certificate is valid for $(G = (V, E), \text{val})$. (15 pts)

3. Provide size parameters for `MineSweep` and use them to describe the big-O number of steps that are required by your verifier from part b using. Why are only two parameters are needed? Conclude that `MineSweep` $\in$ `NP`. (10 pts)