CECS 528, In Class Assignment 7, Spring 2026, Dr. Ebert

Consider the 2SAT instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (\overline{x}_1, \overline{x}_3), (\overline{x}_1, \overline{x}_4), (\overline{x}_1, x_6), (x_2, x_4), (x_3, \overline{x}_6), (\overline{x}_3, x_5), (\overline{x}_5, x_6), (\overline{x}_5, \overline{x}_6)\}.$$

1. Take turns completing a row of the table below by providing two implications that are logically equivalent to the the given binary clause, and then initializing the row. (18 pts)

Clause	Implication	Contrapositive	Initials
$(x_1, \overline{x}_2)$			
$(\overline{x}_1, \overline{x}_3)$			
$(\overline{x}_1, \overline{x}_4)$			
$(\overline{x}_1, x_6)$			
(x_2, x_4)			
$(x_3, \overline{x}_6)$			
$(\overline{x}_3, x_5)$			
$(\overline{x}_5, x_6)$			
$(\overline{x}_5, \overline{x}_6)$			

2. Have one student draw the vertices of the implication graph $G_\mathcal{C}$. Now for each pair of implications in the above table, the student who provided those implications should now add the corresponding edges to $G_\mathcal{C}$. (9 pts)

3. Now take turns computing the necessary reachability sets to determine either a satisfying assignment for $\mathcal{C}$ or to show that $\mathcal{C}$ is unsatisfiable. Proceed in numerical order of variable index with positive before negative: i.e. $R(x_1), R(\bar{x}_1), R(x_2), R(\bar{x}_2), \ldots$. For any valid partial assignments, make sure to redraw the (reduced) implication graph. Hint: for this problem only three reachability sets need to be computed (one for each student). Please initialize the reachability set that you computed. Provide the final satisfying assignment if one exists. (9 pts)

4. Suppose a satisfiable `2SAT` instance $\mathcal{C}$ has an implication graph that includes the path $P = \bar{x}_3, x_1, x_5, x_3$. What can you say about a satisfying assignment α for $\mathcal{C}$? Explain. (4 pts)