CECS 528, In Class Assignment 9, Monday April 13th, 2026, Dr. Ebert

Directions. Read over the problems and decide on who will serve the roles of "Student 1", "Student 2", and "Student 3". Each student will use a separate sheet to write his or her solutions. On the answer sheet, each student should indicate their role (Student 1, Student 2, or Student 3) on the answer sheet. Please turn in your work before the end of class.

Problems

Consider the `3SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x_3}, \overline{x_5}), (x_1, x_3, \overline{x_4}), (x_1, x_2, \overline{x_3}), (x_1, x_3, x_5), (\overline{x_1}, x_2, x_3), (\overline{x_1}, x_2, \overline{x_4}),$$
$$(\overline{x_1}, \overline{x_2}, x_4), (\overline{x_1}, \overline{x_2}, \overline{x_5}), (\overline{x_1}, x_3, x_5), (\overline{x_1}, \overline{x_3}, x_4), (x_2, x_4, \overline{x_5}),$$
$$(\overline{x_2}, \overline{x_3}, x_5), (\overline{x_2}, x_3, \overline{x_4})(x_2, x_3, \overline{x}_4), (x_1, x_2, x_4), (\overline{x}_1, \overline{x}_3, \overline{x}_4)\}.$$

$\mathcal{C}$ is satisfiable and your group will work together to determine a satisfying assignment.

Student 1. Substitute the partial assignment $\alpha_{00} = (x_1 = 0, x_2 = 0)$ into each of the clauses and either deduce the necessary assigned values for x_3, x_4, x_5 that yield a satisfying assignment, or conclude that there is no satisfying assignment that can extend α_{00}. Show work. (5 pts)

Student 2. Substitute the partial assignment $\alpha_{01} = (x_1 = 0, x_2 = 1)$ into each of the clauses and either deduce the necessary assigned values for x_3, x_4, x_5 that yield a satisfying assignment, or conclude that there is no satisfying assignment that can extend α_{01}. Show work. (5 pts)

Student 3. Substitute the partial assignment $\alpha_{10} = (x_1 = 1, x_2 = 0)$ into each of the clauses and either deduce the necessary assigned values for x_3, x_4, x_5 that yield a satisfying assignment, or conclude that there is no satisfying assignment that can extend α_{10}. Show work. (5 pts)

Student 1. For the mapping reduction f : `3SAT` $\rightarrow$ `Clique` provided in lecture, if $f(\mathcal{C}) = (G, k)$, then provide the value of k and the number of vertices and edges of G. Hint: for the number of edges compute the maximum number of possible edges and subtract away all the missing edges due to logical inconsistencies between two vertices who respectively label a variable and its negation. For example, variable x_1 appears as a clause literal 5 times, while $\overline{x}_1$ appears 7 times. Hence, these two literals account for $5 \times 7 = 35$ missing edges. Calculate the number of missing edges caused by the other four variables. (10 pts)

Student 2. Based on Student 1's answer for the clique size k, does G have a k-clique? Why? If yes, then provide the vertices of the clique, making sure to indicate to which vertex group each clique vertex belongs. (5 pts)

Student 3. For the mapping reduction f : `3SAT` $\rightarrow$ `Subset Sum` provided in lecture, if $f(\mathcal{C}) = (S, t)$, then provide the target value t. Does S have a subset that sums to t? Why? If yes, then provide the members of subset A that sums to t. Instead of providing the decimal value of each number, instead provide their names (each of which starts with either y, z, g, or h. Then write the numbers in decimal form, stack them vertically, and verify that their sum does indeed add to t. (10 pts)