CECS 528, Quiz 1, Spring 2026, Friday Dr. Ebert

Directions. Please show all work. Make sure your name is on each solution sheet and number your problems.

Unit 1 LO Problems

LO1. Solve the following.

(a) Use the Master Theorem to determine the big-Θ growth of $T(n)$ if it satisfies

$$T(n) = 9T(n/3) + n^{\log_5(25)}.$$

Solution. $f(n) = n^{\log_5(25)} = n^2$

$n^{\log_3 9} = n^2 \Rightarrow f(n) = \Theta(n^{\log_3 9}) \Rightarrow$

By Case 2 of MT, $T(n) = \Theta(n^2 \log n)$

(b) Use the Substitution method to prove that if

$$T(n) = 4T(n/2) + 3\sqrt{n},$$

Then $T(n) = O(n^2)$. Inductive Assumption:

$T(k) \leq Ck^2 + Dk$ for All $k < n$ and constants $C > 0$ and D.

$T(n) = 4T(n/2) + 3\sqrt{n} \leq 4\left[C\left(\frac{n}{2}\right)^2 + D\left(\frac{n}{2}\right)\right] + 3\sqrt{n}$

$= Cn^2 + 2Dn + 3\sqrt{n} \leq Cn^2 + Dn \iff$

$Dn \leq -3\sqrt{n} \iff D \leq \frac{-3}{\sqrt{n}} \iff$

$D < 0$ and n is sufficiently large. ✓

LO2. Solve the following.

(a) For the `Find Statistic` algorithm describe using **one or two sentences** how the pivot M is obtained for the partitioning step. Also, the inequality

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9$$

was provided to establish that, for the partitioning step, about 30% of the array elements are less than or equal (respectively, greater than or equal) to M. Explain the significance of each of the following constants that make up the left expression: i) $\frac{1}{2}$, ii) -2, iii) 3. Provide a few sentences for each one.

Solution. M = median of group-of-five medians.

$\frac{1}{2}$: $\frac{1}{2}$ the $\lceil \frac{n}{5} \rceil$ group-of-five medians are $\leq M$.

3: For each group-of-five median $\leq M$, there are two other members from its group that are also $\leq M$ $\Rightarrow$ 3 members from that group are $\leq M$. -2: do not count final group and group from which median came.

(b) Draw the recursion tree that results when applying Mergesort to the array

$$a = 5, 4, 12, 8, 7, 11, 13, 9, 10, 16,$$

Label each node with the sub-problem to be solved at that point of the recursion. Assume arrays of size 1 and 2 are base cases. Assume that odd-sized arrays are split so that the left subproblem has one more integer than the right. Next to each node, write the solution to its associated subproblem.