CECS 528, Quiz 1 Solutions, Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions. Please show all work. Make sure your name is on each solution sheet and number your problems.

Unit 1 LO Problems

LO1. Solve the following.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies

$$T(n) = 9T(n/3) + n^2 \log^3 n.$$

Solution. $n^{\log_3 9} = n^2 \quad f(n) = n^2 \log^3 n$

$\therefore$ By case 4 of M. T., $T(n) = \Theta(n^2 \log^4 n)$

(b) Use the Substitution method to prove that if

$$T(n) = 4T(n/2) + \log^2 n,$$

Then $T(n) = O(n^2)$.

Solution. Inductive assumption: $T(k) \le Ck^2 + Dk$, for all $k < n$, some const. $C > 0$ and some const. D.

$T(n) \le 4\left[C\left(\frac{n}{2}\right)^2 + D\left(\frac{n}{2}\right)\right] + \log^2 n = Cn^2 + 2Dn + \log^2 n \le Cn^2 + Dn \Leftrightarrow Dn + \log^2 n \le 0 \Leftrightarrow Dn \le -\log^2 n \Leftrightarrow D \le -\frac{\log^2 n}{n}$ which is true for $D \le -1$ and n sufficiently large.

LO2. Solve the following.

(a) Consider Karatsuba's algorithm which we'll call `multiply` for multiplying two even-length n-bit binary numbers x and y. Let x_L and x_R be the leftmost $n/2$ and rightmost $n/2$ bits of x respectively. Define y_L and y_R similarly. Let P_1 be the result of calling `multiply` on inputs x_L and y_L, P_2 be the result of calling `multiply` on inputs x_R and y_R, and P_3 the result of calling `multiply` on inputs $x_L + x_R$ and $y_L + y_R$. Then return the value

$$P_1 \times 2^n + (P_3 - P_1 - P_2) \times 2^{n/2} + P_2.$$

Prove that the returned value does in fact equal xy. Hint: first provide an arithmetc expression that expresses x (repectively, y) in terms of x_L and x_R (respectively, y_L and y_R).

Solution.

$xy = (2^{n/2}x_L + x_R)(2^{n/2}y_L + y_R) = 2^n x_L y_L + x_R y_R + 2^{n/2}(x_R y_L + y_R x_L) = P_1 2^n + P_2 + 2^{n/2}(P_3 - P_1 - P_2)$, since $P_3 - P_1 - P_2 = x_L y_L + x_R y_R + x_R y_L + y_R x_L - x_L y_L - x_R y_R = x_R y_L + y_R x_L$

(b) Demonstrate Hoare's `Quicksort` algorithm on the array

$$a = \underline{67}, 25, 61, 77, 66, 13, \underline{73}, 70, 81, 17, 74, 39, \underline{40}.$$

Hint: it is *unncessary* to rewrite the array each time the left and right markers are moved.

Solution. median(67, 73, 40) = 67

~~67~~ 40, 25, 61, ~~77~~ 39, 66, 13, ~~73~~ 17, ~~70~~ 67, 81, ~~17~~ 73, 74, ~~39~~ 77, ~~40~~ ~~67~~ 70

l_1 l_2 l_3 r_1 r_2 r_3 M

a_{left} = 40 25 61 39 66 13 17

a_{right} = 81, 73, 74, 77, 70