CECS 528, Quiz 1, Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions. Please show all work. Make sure your name is on each solution sheet and number your problems.

Unit 1 LO Problems

LO1. Solve the following.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies

$$T(n) = 9T(n/3) + n^2 \log^3 n.$$

(b) Use the Substitution method to prove that if

$$T(n) = 4T(n/2) + \log^2 n,$$

Then $T(n) = \mathrm{O}(n^2)$.

LO2. Solve the following.

(a) Consider Karatsuba's algorithm which we'll call `multiply` for multiplying two even-length n-bit binary numbers x and y. Let x_L and x_R be the leftmost $n/2$ and rightmost $n/2$ bits of x respectively. Define y_L and y_R similarly. Let P_1 be the result of calling `multiply` on inputs x_L and y_L, P_2 be the result of calling `multiply` on inputs x_R and y_R, and P_3 the result of calling `multiply` on inputs $x_L + x_R$ and $y_L + y_R$. Then return the value

$$P_1 \times 2^n + (P_3 - P_1 - P_2) \times 2^{n/2} + P_2.$$

Prove that the returned value does in fact equal xy. Hint: first provide an arithmetc expression that expresses x (repectively, y) in terms of x_L and x_R (respectively, y_L and y_R).

(b) Demonstrate Hoare's `Quicksort` algorithm on the array

$$a = 67, 25, 61, 77, 66, 13, 73, 70, 81, 17, 74, 39, 40.$$

Hint: it is *unncessary* to rewrite the array each time the left and right markers are moved.